APPLICATION OF DERIVATIVES

1) The slope of the tangent to the curve represented by $x = t^2+3t-8$ and $y = 2t^2-2t-5$ at the point M(2,-1) is

(a) $7/6$ (b) $2/3$ (c) $3/2$ (d) $6/7$ Ans (d)

2) The function $f(x) = 2 \log (x-2) - x^2+4x+1$ increases in the interval.

(a) (1,2) (b) (2,3) (c) $\left(5/2, 3\right)$ (d) (2,4) Ans: (b) and (c)

3) The function $y = \tan^{-1}x - x$ decreases in the interval of

(a) $(1, \infty)$ (b) $(-1, \infty)$ (c) $(-\infty, \infty)$ (d) $(0, \infty)$ Ans: all

4) The value of a for which the function $f(x) = a\text{Sin}x + \frac{1}{3}\text{Sin}3x$ has an extreme at $x = \Pi/3$ is

(a) 1 (b) −1 (c) (O) (d) 2 Ans: d

5) The co-ordinates of the point p(x,y) in the first quadrant on the ellipse $\frac{x^2}{8}+\frac{y^2}{18}=1$ so that the area of the triangle formed by the tangent at P and the co-ordinate axes is the smallest are given by

(a) (2,3) (b) $\sqrt{8},0$ (d) $\left(\sqrt{18},0\right)$ (d) none of these Ans: (a)

6) The difference between the greatest and the least values of the function

$f(x) = \cos x + \frac{1}{2}\cos 2x - \frac{1}{3}\cos 3x$ is

(a) $2/3$ (b) $8/7$ (c) $9/4$ (d) $3/8$ Ans: (c)

7) If $y = a \log|x| + bx^2+x$ has its extreme values at x = −1 and x = 2 then

(a) a=2 b= −1 (b) a=2, b = −½ (c) a=−2, b=½ (d) none of these Ans: (b)

8) If θ is the semivertical angle of a cone of maximum volume and given slant height, then $\tan\theta$ is given by

(a) 2 (b) 1 (c) $\sqrt{2}$ (d) $\sqrt{3}$ Ans: (c)

9) If $f(x) = \frac{x}{\sin x}$ and $g(x) = \frac{x}{\tan x}$ where $0 \le x \le 1$ then in this interval

(a) both f(x) and g(x) are increasing
(b) both f(x) and g(x) are decreasing
(c) f(x) is an increasing function
(d) g(x) is an increasing function Ans: (c)

10) If $f(x) = \begin{cases} 3x^2+12x-1 & : -1 \le x \le 2 \\ 37-x & : 2 < x \le 3 \end{cases}$ then

(a) f(x) is increasing on [−1,2]
(b) f(x) is continuous on [−1,3]
(c) $f^{|}(2)$ doesn't exist
(d) f(x) has the maximum value at x = 2 Ans: (a),(b),(c),(d)

11) The function $\frac{\sin(x+\alpha)}{\sin(x+\beta)}$ has no maximum or minimum value if

(a) $\beta-\alpha = k\Pi$ (b) $\beta - \alpha \ne k\Pi$ (c) $\beta - \alpha = 2k\Pi$
(d) None of these where K is an integer. Ans: (b)

12) If $f(x)=\frac{x^2-1}{x^2+1}$ for every real number then minimum value of f

(a) does not exist (b) is not attained even though f is bounded
(c) is equal to 1 (d) is equal to -1 Ans: (d)

13) If the line ax+by+c = 0 is normal to the curve xy = 1 then

(a) a>0, b>0 (b) a>0, b<0 (d) a<0, b>0 (d) a<0, b<0 Ans: (b),(c)

14) The tangent to the curve
$x = a\sqrt{\cos 2\theta}\cos\theta \quad y = a\sqrt{\cos 2\theta}\sin\theta$ at the point corresponding to $\theta = \Pi/6$ is

(a) Parallel to the x-axis (b) Parallel to the y-axis
(c) Parallel to the line y = x (d) none of these Ans: (a)

15) The minimum value of f(x) = |3—x| + |2+x| + |5—x| is

(a) 0 (b) 7 (c) 8 (d) 10 Ans: (b)

16) If $x = a(\theta - \sin\theta),\ \ y = a(1 - \cos\theta)$ then $\frac{d^2y}{dx^2}$ at $\theta = \Pi/2$ is

(a) $\frac{1}{a}$ (b) $\frac{1}{2}$ (c) $-\frac{1}{a}$ (d) $-\frac{1}{2a}$ Ans: (c)

17) If $y = \log\tan\left(\frac{\Pi}{4}+\frac{\Pi}{2}\right)$ then $\frac{dy}{dx}$ is

(a) 0 (b) cosx (c) —secx (d) secx Ans: (d)

18) 'C' on LMV for f(x) = x^2—3x in [0,1] is

(a) 0 (b) $\frac{1}{2}$ (c) $-\frac{1}{2}$ (d) does not exist Ans: (b)

19) The values of a and b for which the function $f(x)=\begin{cases} ax+1 & x \le 3 \\ bx+3 & x > 3 \end{cases}$ is continuous at x = 3 are

(a) 3a + 2b = 5 (b) 3a = 2+3b (c) 3,2 (d) none of these Ans: (b)

20) The tangents to the curve y = x^3+6 at the points (-1,5) and (1,7) are

(a) Perpendicular (b) parallel (c) coincident (d) none of these Ans: (b)

21) If $\frac{dy}{dx}=0$ then the tangent is

(a) Parallel to x-axis (b) parallel to y-axis (c) Perpendicular to x-axis
(d) perpendicular to y-axis Ans: (a)

22) If the slope of the tangent is zero at (x_1, y_1) then the equation of the tangent at (x_1,y_1) is

(a) y_1=mx_1+c (b) y_1= mx_1 (c) y—y_1 (d) y=0 Ans: (c)

23) The function $f(x) = -\frac{x}{2}+\sin x$ is always increasing in

(a) $\left(\frac{-\Pi}{2},\frac{\Pi}{2}\right)$ (b) $\left(0,\frac{\Pi}{4}\right)$ (c) $\left(\frac{\Pi}{4},\frac{\Pi}{2}\right)$ (d) $\left(\frac{-\Pi}{3},\frac{\Pi}{3}\right)$ Ans: (d)\

24) The least value of 'a' such that the function f(x) = x^2+ax+1 is strictly increasing on (1,2) is

(a) —2 (b) 2 (c) $\frac{1}{2}$ (d) $-\frac{1}{2}$ Ans: (a)

25) The least value of f(x) = tan-1 (sinx + cosx) strictly increasing is

(a) $\left(\frac{\Pi}{4},\frac{\Pi}{2}\right)$ (b) $\left(0,\frac{\Pi}{2}\right)$ (c) 0 (d) none of these Ans: (d)

4-6 Marks

26) Determine the points of maxima and minima of the function

$f(x)=\frac{1}{8}\log x - bx + cx^2$ where $b \geq 0$

Ans: f has maxima at $2 = \frac{1}{4}\left(b - \sqrt{b^2 - 1}\right)$ and minima at $\beta = \frac{1}{4}\left(b + \sqrt{b^2-1}\right)$

27) Find the interval in which the following functions are increasing or decreasing

(a) $y = \log\left(x + \sqrt{1+x^2}\right)$ (b) $y = \frac{10}{4x^3 - 9x^2 + 6x}$

Ans: (a) increases on $(-\infty,\infty)$ (b) increases on $\left(\frac{1}{2},1\right)$ and decreases on $(-\infty,0)\cup\left(0,\frac{1}{2}\right)\cup(1,\infty)$

28) Find the equation of normal to the curve $y = (1+x)^y + \sin^{-1}(\sin^2 x)$ at $x = 0$ Ans: x+y=1

29) If P_1 and P_2 are the lengths of the perpendiculars from origin on the tangent and normal to the curve $x^{2/3} + y^{2/3} = a^{2/3}$ respectively Prove that $4P_1^2 + P_2^2 = a^2$

30) What angle is formed by the y-axis and the tangent to the parabola $y = x^2+4x-17$ at the point $P\left(5/2, -3/4\right)$? Ans: $\theta = \Pi/2 - \tan^{-1} 9$

31) A cone is circumscribed about a sphere of radius r. Show that the volume of the cone is maximum when its semi vertical angle is $\sin^{-1}\left(\frac{1}{3}\right)$

32) Find the interval in which the function f(x) is increasing or decreasing

$f(x) = x^3 - 12x^2 + 36x + 17$ Ans: Increasing in x<2 or x>6

Decreasing 2<x<6

33) Find the equation of tangent to the curve $\sqrt{x} + \sqrt{y} = \sqrt{a}$ at the point (x_1, y_1) and show that the sum is intercepts on the axes is constant.

34) Find the point on the curve $y = x^3-11x+5$ at which the tangent has equation $y = x-11$

Ans: (2,—9)

35) Find the equation of all lines having slope —1 and that are tangents to the curve

$y = \frac{1}{x-1}, x \neq 1$ Ans: x+y+1=0 ; x+y—3=0

36) Prove that the curves $y^2 = 4ax$ and $xy = c^2$ cut at right angles if $c^4 = 32a^4$

37) Find the points on the curve $y = x^3 - 3x^2+2x$ at which the tangent to the curve is parallel to the line $y - 2x + 3 = 0$. Ans: (0,0) (2,0)

38) Show that the semi vertical angle of a right circular cone of maximum volume and given slant height is $\tan^{-1}\sqrt{2}$

39) Show that the semi vertical angle of a right circular cone of given total surface area and maximum volume is $\sin^{-1}\frac{1}{3}$

40) Show that the right circular cone of least curved surface area and given volume is $\sqrt{2}$ times the radius of the base.

41) Show that the height of a cylinder of maximum volume that can be inscribed in a sphere of radius R is $\frac{2R}{\sqrt{3}}$

42) Find the absolute maximum and absolute minimum value of $f(x) = 2\cos x + x \quad x \varepsilon [0, \Pi]$

Ans: max at $x = \Pi/6$ and min at $x = 5\Pi/6$

43) Show that the volume of greatest cylinder which can be inscribed in a cone of height h and semi vertical angle α is $\frac{4}{27}\Pi h^3 \tan^2 \alpha$

44) A window is in the form of a rectangle above which there is a semicircle. If the perimeter of the window is 'P' cm. Show that the window will allow the maximum possible light only when the radius of the semi circle is $\frac{P}{\Pi + 4}$cm

45) Find the area of the greatest isoscles triangle that can be inscribed in a given ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ with its vertex coinciding with one extremity of the major axis.

46) A rectangular window is surmounted by an equilateral triangle. Given that the perimeter is 16 cm. Find the width of the window so that the maximum amount of light may enter.

47) Show that the volume of the greatest cylinder which can be inscribed in a cone of height h and semi vertical angle 30° is $\frac{4}{81}\Pi h^3$

48) Show that the height of the right circular cylinder of maximum volume that can be inscribed in a given right circular cone of height h is $h/3$.

49) Of all the rectangles each of which has perimeter 40 metres find one which has maximum area. Find the area also.

50) Show that the rectangle of maximum area that can be inscribed in a circle of radius 'r' cms is a square of side $\sqrt{2}\, r$

51) Prove that the volume of the largest cone that can be inscribed in a sphere of radius R is $\frac{8}{27}$ of the volume of the sphere.

52) Show that the semi vertical angle of the cone of maximum volume and of given slant height is $\tan^{-1}\sqrt{2}$

53) Given the sum of the perimeters of a square and a circle show that the sum of their areas is least when the side of the square is equal to the diameter of a circle.

54) Find the maximum slope of the curve $f(x) = 2x + 3x^2 - x^3 - 27$

55) Show that the height of the cone of maximum volume that can be inscribed in a sphere of radius 12 cm is 16 cm.

56) A point on the hypotenuse of a right angled triangle is at distances a and b from the sides. Show that the length of the hypotenuse is at least $\left(a^{2/3} + b^{2/3}\right)^{3/2}$

57) Sand is pouring from a pipe at the rate of $12cm^3/sec$. The falling sand forms a cone on the ground in such a way that the height of the cone is always one-sixth of the radius of the base. How fast is the height of the sand cone in increasing when the height is 4 cm?

Ans : $\frac{1}{48\Pi} cm/sec.$

58) A man 160 cm tall walks away from a source of light situated at the top of the pole 6m high at the rate of 1.1 m/sec. How fast is the length of the shadow increasing when he is 1m away from the pole. Ans: 0.4 cm/sec.

59) An edge of a variable cube is increasing at the rate of 5 cm/sec. How fast is the volume of the cube is increasing when edge is 10cm long? Ans: $1500 cm^3/sec$.

60) A balloon which always remains spherical is being inflated by pumping in gas at the rate of 900 cm^3/sec. Find the rate at which the radius of the balloon is increasing when the radius of the balloon is 15 cm. Ans: $\frac{1}{\Pi} cm/sec.$

61) The volume of a spherical balloon is increasing at the rate of 25 cm^3/sec. Find the rate of change of its surface area at the instant when the radius is 5 cm. Ans: 10 cm^2/sec.

62) The surface area of a spherical bubble is increasing at the rate of 2 cm^2/sec. Find the rate at which the volume of the bubble is increasing at the instant if its radius is 6 cm.

Ans: $80\Pi cm^2/sec.$

63) Gas is escaping from a spherical balloon at the rate of 900 cm^3/sec. How fast is the surface area, radius of the balloon shrinking when the radius of the balloon is 30cm?

Ans: $\frac{dA}{dt} = 60cm^2/sec.\ \frac{dr}{dt} = \frac{1}{4\Pi} cm/sec.$

64) Water is passed into an inverted cone of base radius 5 cm and depth 10 cm at the rate of $\frac{3}{2} c.c./sec.$ Find the rate at which the level of water is rising when depth is 4 cm.

Ans: $\frac{3}{8\Pi} cm/sec.$

65) Show that the function $f(x) = e^{2x}$ is strictly increasing on R.

66) Show that $f(x) = 3x^5 + 40x^3 + 240x$ is always increasing on R.

67) Find the interval in which the function $f(x) = x^4-4x^3+4x^2+15$ is increasing or decreasing.

68) Find whether $f(x) = \cos(2x + \Pi/4)\ \frac{3\Pi}{8} < x < \frac{5\Pi}{8}$ is increasing or decreasing.

69) Find the interval in which the function $\frac{4.\sin x - 2x - x\cos x}{2 + \cos x}$ is increasing or decreasing.

Ans: $(0, \Pi/2) \cup \left(\frac{3\Pi}{2}, 2\Pi\right)$

70) FInd the interval in which $f(x) = 8+36x+3x^2-2x^3$ is increasing or decreasing.

Ans: Increasing : $2 < x < 3$ decreasing : $x > 3$ or $x < -2$

71) Find the interval on which the function $\frac{x}{\log x}$ is increasing or decreasing.

Ans: Increasing in $((e, \infty)$

decreasing for (0,1)U(1,e)

72) Find the points on the curve $\frac{x^2}{4}+\frac{y^2}{25}=1$ at which the tangents are parallel to x-axis and parallel to y-axis. Ans: $(0,\pm5), (\pm2,0)$

73) Find equation of tangents to the curve $x = a\cos^3\theta,\ y = b\sin^3\theta$ at $\theta = \Pi/4$

74) Find the equations of the normal lines to the curve $y = 4x^3-3x+5$ which are parallel to the line $9y+x+3 = 0$. Ans: $x+9y-55=0,\ x+9y-35=0$

75) FInd the equation of tangent and normal to the curve $y^2 = \frac{x^3}{4-x}$ at $(2, -2)$

Ans: $2x+y-2=0,\ x-2y-6=0$

76) FInd the equation of the tangent to the curve $\sqrt{x}+\sqrt{y} = a$ at the point $\left(\frac{a^2}{4},\frac{a^2}{4}\right)$

Ans: $2x+2y=a^2$

77) Find the angle between the parabolas $y^2=4ax$ and $x^2=4by$ at their point of intersection other than the origin. Ans: $\theta = \tan^{-1}\left[\frac{3a^{1/3}b^{1/3}}{2\left(a^{2/3}+b^{2/3}\right)}\right]$

78) Using differentials find the appropriate value of $(82)^{1/4}$ Ans: 3.0092

79) If $y = x^4-10$ and if x changes from 2 to 1.97, what is the appropriate change in y ?

Ans: -0.96, y changes from 6 to 5.04

80) Find the appropriate change in volume of a cube when side increases by 1%. Ans: 3%

81) Use differentials to evaluate $\left(\frac{17}{81}\right)^{1/4}$ approximately. Ans: 0.677

82) Using differentials evaluate tan 44° approximately, 1° = 0.07145°. Ans: 0.9651

83) Find the approximate value of x if $2x^4-160 = 0$ Ans: 2.991

84) Find the maximum and minimum values of 'f', if any of the function $f(x) = |x|,\ x \varepsilon R$

85) Find the maximum and minimum value of $f(x) = |(\sin 4x+5)|$ without using derivatives.

86) The curve $y = ax^3+6x^2+bx+5$ touches the x-axis at P(-2,0) and cuts the y-axis at a point Q where its gradient is 3. Find a,b,c.

Ans: $a = -\frac{1}{2}\ b = \frac{-3}{4},\ c = 3$

87) FInd the local maxima and local minima if any of the function $f(x) = e^{5x}$

88) Find the maxima or minima if any of the function $f(x) = \frac{1}{x^2+2}$

Ans: local max at x = 0, value $\frac{1}{2}$

89) Without using derivatives find the maximum or minimum value of $f(x) = -|x+5| + 3$

Ans: max value 3, no minimum value

90) Without using derivatives find the maximum and minimum value of $f(x) = \sin 2x + 7$

Ans: Max. value 8, min. value 6

91) FInd whether $f(x) = e^x$ has maxima or minima. Ans: No maxima nor minima

92) At what point in the interval $[0,2\Pi]$ does the function Sin2x attain its maximum value?

93) Find the intervals in which $f(x) = \log\cos x$ is strictly decreasing and strictly increasing.

Ans: decreasing $\left(0,\Pi/2\right)$, increasing $\left(\Pi/2,\Pi\right)$

94) Find the interval in which $y = x^2e^{-x}$ is increasing. Ans: (0,2)

95) Find two positive numbers x and y such that their sum is 16 and sum of whose cubes is minimum.

Ans: 8,8

96) Find the local maximum and local minimum value of the function.

$f(x) = \sin x + \frac{1}{2}\cos 2x$ in $\left[0,\Pi/2\right]$ Ans: local max. value $\frac{3}{4}$ at $x = \frac{\Pi}{6}$

97) Two equal sides of an isosceles triangle with fixed base b are decreasing at the rate of 3cm/sec. How fast is the area decreasing when the two equal sides are equal to the base?

98) A poster is to contain $50cm^2$ of matter with borders of 4 cm at top and bottom and of 2 cm on each side. Find the dimensions if the total area of the poster is minimum.

99) Find the sides of a rectangle of greatest area that can be inscribed in the ellipse $x^2 + 4y^2 = 16$

Ans: $4\sqrt{2}, 2\sqrt{2}$

100) Find the maximum profit that a company can make if the profit function is given by

$P(x) = 41 - 24x - 18x^2$ Ans: 49

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